

# Math1105 Bonus Problem 1

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Below is your first Math1105 bonus problems. To receive credit for your work, you must come to office hours to demonstrate your progress toward a solution to both problems. You may use regular office hours from 1-2pm on Mondays and Fridays or you can make an appointment via email. Our meeting must occur by Monday Oct. 6.

1. Suppose that  $f(x)$  is a function that, for a real number  $L$ , satisfies

$$\lim_{x \rightarrow 0^+} f(x) = L.$$

Is it the case that  $L$  is the only such number?

(Hint): Suppose there are 2 real numbers  $L \neq M$  so that

1.  $\lim_{x \rightarrow 0^+} f(x) = L$ , and

2.  $\lim_{x \rightarrow 0^+} f(x) = M$ .

Now explain why this cannot happen.

2. Draw the graph of a function  $g(x)$  that satisfies each of the following properties.

- (a)  $\lim_{x \rightarrow -\infty} g(x) = -4$ .

- (b)  $\lim_{x \rightarrow -2^-} g(x) = -2$ .

- (c)  $\lim_{x \rightarrow -2^+} g(x) = 2$ .

- (d)  $g(x)$  is not differentiable at  $x = 0$ .

- (e)  $\lim_{x \rightarrow 1^-} g(x) = \infty$ .

- (f)  $\lim_{x \rightarrow 1^+} g(x) = -\infty$ .

- (g)  $\lim_{x \rightarrow 2} g(x)$  does not exist.

- (h)  $\lim_{x \rightarrow \infty} g(x) = 4$ .