Math1105 Midterm Test - Sample

Cape Breton University Mathematics

November 10, 2025

This is a closed book test. There are **8** problems.The value of each problem is given in brackets before each problem statement. You may not use notes of any kind nor communicate with any persons with the exceptions of Dr. Rodney and his assistant(s). Note that the CBU policy concerning academic dishonesty will be enforced. Communication devices are prohibited as are graphing calculators. You are permitted to use a standard scientific calculator. If you have a question, raise your hand and someone will be with you shortly.

Good Luck!

1. **True/False Problems** Circle **T** for true and **F** for false. You do not need to show your work. Each item is worth 1 mark.

(a)	If $f(x)$ is continuous and differentiable on $[a, b]$ and there is a $c \in [a, b]$ where $f'(x) = 0$ then $f(a) = f(b)$.	**T**	**F**
(b)	If $f(x) = \cos(x)$ then $f^{(713)}(x) = \sin(x)$	**T**	**F**
(c)	There are two different numbers a, b such that $\arcsin(a) = \arcsin(b)$	**T**	**F**
(d)	There is more than one function that satisfies the differential equation $\frac{dy}{dx} = Ky$ for some fixed constant $K \in \mathbb{R}$.	**T**	**F**

2. **Some differentiation**. Find the derivative of each of the following. Show all of your work.

   (a) $f(x) = \sec(x)e^{2x}$

   (b) $g(t) = \sqrt{-\ln(\cos(t))}$

   (c) $h(x) = \frac{\arctan(x)}{x+1}$.

3. **Fun with the Mean Value Theorem**. Let $f(x) = x^2 \sin(\pi x) + (1 - x)$

   (a) Explain why you can use the Mean Value Theorem on $f(x)$ over any interval $[a, b]$.

   (b) Show that there is at least one real number c such that $f'(x) = 0$.

   (c) Sketch a continuous function $g(x)$ over the interval $[-1, 1]$ where $g(a) = g(b)$ but where there is no $c \in [-1, 1]$ where $g\prime(c) = 0$. Why doesn't the Mean Value Theorem apply to your function?

4. **Implicit Differentiation**.

   (i) Find the location of all horizontal tangents to the curve defined by

$$x^2 + 4x + 4y^2 = 12.$$

   (ii) (4) Use implicit differentiation to show that

$$\frac{d}{dt}\sin^{-1}(t) = \frac{1}{\sqrt{1 - t^2}}.$$

5. **Higher Derivatives** Let $s(t) = bt^3 + ct^2 + dt + e$ be the position of a particle along the x-axis at time t, for some $b, c, d.e \in \mathbb{R}$. Recall that $s'(t)$ is the velocity $v(t)$, $v'(t) = a(t)$ is the acceleration, and $a'(t) = j(t)$ is the jerk of the particle at time t.

   (a) Suppose you know that $s(1) = 1, v(1) = 3$, $a(1) = -6$, and $j(1) = 12$. Determine b, c, d, e in the function $s(t)$.

   (b) Determine at which times the particle's velocity is increasing and at which times the particle's velocity is decreasing.

6. **Logarithmic Differentiation**

   (a) Using logarithmic differentiation find $\frac{d}{dx}\left((x + 1)(x^2 + 1)^2(x^3 + 1)^3(x^4 + 1)^4(x^5 + 1)^5\right)$

(b) (3) Find $f'(x)$ if $f(x) = (x+1)^{(x+1)}$. Explain why the power rule and the rule for derivatives of exponential functions does not apply.

7. **Fun with Exponential Decay** In 2020, a physics lab had a 25kg sample of a radioactive element. However, its label is missing. You know that 5 years later 19.61kg of the sample remain. Due to other chemical properties, the lab has narrowed the possibilities down to Plutonium-241 (half-life 14.29 years), Barium-133 (half-life 10.74 years), and Curium-244 (half-life 18.10 years)
   (a) Determine the which element the sample is made from.
   (b) Given your answer in part a, how many years until the sample is reduced to a size of 1kg?
8. **Related Rates** Two tall sticks are vertically planted into the ground, separated by a distance of 30 cm. We simultaneously put two snails at the base of each stick. The two snails then begin to climb their respective sticks. The first snail is moving with a speed of 25 cm per minute, while the second snail is moving with a speed of 15 cm per minute. What is the rate of change of the distance between the two snails when the first snail reaches 100 cm above the ground?
