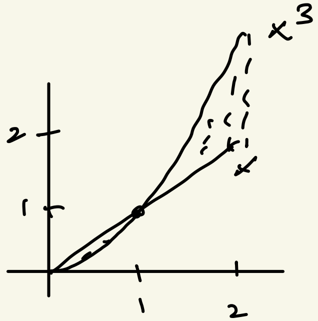
Homework #6
Sol^n.

#1. (a)



$$A = \int_0^2 |x - x^3|\,dx = \int_0^1 (x - x^3)\,dx + \int_1^2 (x^3 - x)\,dx$$

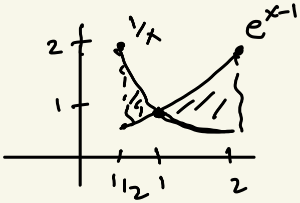
$$= \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1 + \left(\frac{x^4}{4} - \frac{x^2}{2}\Big|_1^2\right)$$

$$= \frac{1}{2} - \frac{1}{4} + 4 - 2 - \frac{1}{4} + \frac{1}{2}.$$

$$= 3 - 1/2$$

$$= 5/2 \text{ sq. units.}$$

(b)



* $e^0 = 1 \longrightarrow e^{x-1}$ is $\uparrow$ on $(1/2, 1)$

$1/x$ is $\searrow$ on $(1/2, 1)$ so they only intersect once.

at $x = 1$, $e^{x-1} = 1 = 1/x$

$$A = \int_{1/2}^2 \left|e^{x-1} - \frac{1}{x}\right| dx = \int_{1/2}^1 \left(\frac{1}{x} - e^{x-1}\right)dx + \int_1^2 \left(e^{x-1} - \frac{1}{x}\right)dx$$

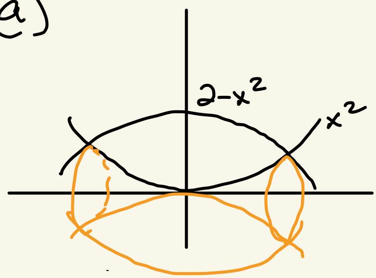
$$= \ln x - e^{x-1}\Big|_{1/2}^{1} + \left(e^{x-1} - \ln x\right)\Big|_{1}^{2}$$

$$= 0 - 1 - \ln\left(\tfrac{1}{2}\right) + e^{-1/2}$$

$$+ e - \ln(2) - 1 + 0$$

$$= e + e^{-1/2} - 2$$

#2. (a)



$2-x^2 = x^2$ if $2x^2 = 2$

giving $x = \pm 1$

$$\text{Vol} = \pi \int_{-1}^{1} (R^2 - r^2)\,dx$$

$$= \pi \int_{-1}^{1} \left[(2-x^2)^2 - (x^2)^2\right] dx$$

$$= \pi \int_{-1}^{1} (4 - 4x^2 + x^4 - x^4)\,dx$$

$$= \pi \int_{-1}^{1} (4 - 4x^2)\,dx$$

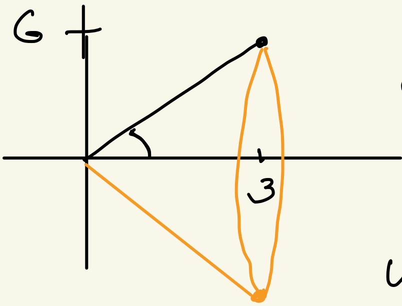
$$= \pi \left(4x - \frac{4}{3}x^3 \Big|_{-1}^{1}\right) =$$

$$= \pi\left(4 - \frac{4}{3} + 4 - \frac{4}{3}\right)$$

$$= \pi\left(8 - \frac{8}{3}\right)$$

$$= \frac{16}{3}\pi \text{ cubic units.}$$

(b) $f(x) = 2x$ over $(0,3)$.



It's a circular cone with height $h=3$ and base radius $r=6$.

We expect $V = \frac{1}{3}\pi r^2 h$

$= 36\pi$

By washers:

$$V = \pi\int_0^3 (2x)^2 dx = \frac{4}{3}\pi x^3 \Big|_0^3$$

$$= \frac{4}{3}\pi \cdot 3^3$$

$$= 4 \cdot 9 \cdot \pi = 36\pi.$$

#3. $V = \lim_{c\to\infty} \pi \int_1^c \frac{1}{x^2}\,dx$

$$= \lim_{c\to\infty} -\frac{\pi}{x}\Big|_1^c$$

$$= \lim_{c\to\infty}\left(\pi - \frac{\pi}{c}\right)$$

$$= \pi \text{ cubic units.}$$

The surface area is ∞!