Math 1206 HW#1

Solutions.

#1. (i) $\sum_{n=3}^{5} \frac{1}{n} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

$= \frac{7}{12} + \frac{1}{5}$

$= \frac{47}{60}$

(ii) $\sum_{m=1}^{36} (m+2)^2$

$= \sum_{m=1}^{36} m^2 + 4\sum_{m=1}^{36} m + 4\sum_{m=1}^{36} 1$

uses

$\sum_{m=1}^{n} m^2 = \frac{1}{6} n(n+1)(2n+1)$

$\sum_{m=1}^{n} m = \frac{1}{2} n(n+1)$

$\sum_{m=1}^{n} 1 = n.$

$= \frac{1}{6}(36)(37)(73) + 2(36)(37) + 4(36)$

$= 19{,}014$

<u>OR</u> $\sum_{m=1}^{36} (m+2)^2 = \sum_{m=3}^{38} m^2 = \sum_{m=1}^{38} m^2 - 5$

$= \frac{1}{6} 38(39)(77) - 5$

$= 19{,}014.$

(iii) $\sum_{k=1}^{75}\left((k+2)^2-(k+1)^2\right)$

This is a telescoping series.

That is,

$$\sum_{k=1}^{n}\left[(k+2)^2-(k+1)^2\right] = \left[\cancel{3^2}-2^2\right]+\left[\cancel{4^2}-\cancel{3^2}\right]+\dots+\left[(n+2)^2-\cancel{(n+1)^2}\right]$$

$$= (n+2)^2-4$$

So, when $n=75$, our sum is $77^2-4 = 5925$.

#2. $S_k = \sum_{n=1}^{k} p^n$

(i). $pS_1 - S_1$: $S_1 = \sum_{n=1}^{1} p^n = p$

So $pS_1 - S_1 = p^2 - p = p(p-1)$

- $pS_{10}-S_{10}$: $S_{10} = p+p^2+\dots+p^{10}$

$$pS_{10} = p^2+p^3+\dots+p^{11}$$

So $pS_{10}-S_{10} = p^{11}-p = p(p^{10}-1)$

(ii) $S_k = p + p^2 + \dots p^k + p^k$

$pS_k = p^2 + p^3 + \dots + p^k + p^{k+1}$

we get $pS_k - S_k = p^{k+1} - p$

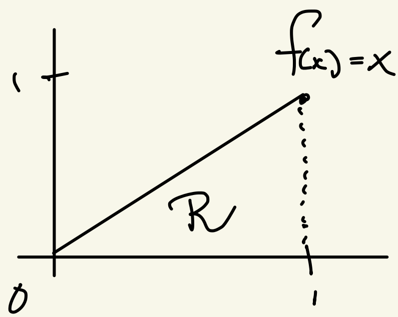
$= p(p^k - 1)$

i.e. $pS_k - S_k = p(p^k - 1)$

finding S_k we get

$$S_k = \frac{p(p^k - 1)}{p-1}$$

#3.



We want the area of R.

(i) $x_0 = 0,\ x_1 = \frac{1}{n},\ x_2 = \frac{2}{n}, \dots, x_n = 1$

Bases of Rectangles are

$$[0, 1/n], [\tfrac{1}{n}, \tfrac{2}{n}], \dots, [\tfrac{n-1}{n}, 1]$$

all of width $\Delta x = 1/n$.

Heights are taken as $f(\frac{j}{n})$ where $\frac{j}{n}$ is the right end point of the j^{TH} interval $[x_{j-1}, x_j]$.

So
$$\sum_{j=1}^{n} A(R_j) = \sum_{j=1}^{n} f(\tfrac{j}{n}) \Delta x$$
$$= \sum_{j=1}^{n} \frac{j}{n} \cdot \frac{1}{n}$$
$$= \frac{1}{n^2} \sum_{j=1}^{n} j$$
$$= \frac{1}{2n^2}(n)(n+1) = \frac{n(n+1)}{2n^2}.$$

(ii) $$\lim_{n\to\infty} \sum_{j=1}^{n} A(R_j) = \lim_{n\to\infty} \frac{n(n+1)}{2n^2} = \frac{1}{2}.$$

(iii) R is a triangle with area

$$A(R) = \frac{1}{2} b \times h = \frac{1}{2}(1)(1) = 1/2.$$

<u>it's the same!</u>